

Whereas our distillation procedure employs only local operations (filtering) and classical communication (coincidence counting), our non-locality demonstrations clearly rely on ‘conditional probabilities’^{16,25}—we include only events in which both photons are transmitted by their respective partial polarizers. Nevertheless, we emphasize that these filters are before the analysers, and are thus independent of the polarization measurements. The filtering process is used to select a particular subensemble of the original states. The entanglement for members of this subensemble (corresponding to coincident detections) is higher than the average for the entire ensemble (which includes photons reflected at the partial polarizers—these possess no entanglement whatsoever), and therefore they are able to demonstrate non-local effects, such as violations of Bell’s inequalities. This is completely consistent with the fact that LOCC cannot be used to increase the average entanglement of an ensemble⁴. For instance, for our second trial, the initial degree of entanglement (of formation)²⁶ before filtering is $E = 0.32$; after filtering the transmitted photons have $E = 0.44$, but the reflected photons have $E = 0$. □

Received 11 September; accepted 15 December 2000.

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Acknowledgements

We thank H. Bernstein, D. DiVincenzo, B.-G. Englert, L. Hardy, D. James and M. Zukowski for helpful discussions. This work was supported in part by the National Security Agency (NSA) and Advanced Research and Development Activity (ARDA), and by the European IST FET QuComm project.

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Fluid particle accelerations in fully developed turbulence

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The motion of fluid particles as they are pushed along erratic trajectories by fluctuating pressure gradients is fundamental to transport and mixing in turbulence. It is essential in cloud formation and atmospheric transport^{1,2}, processes in stirred chemical reactors and combustion systems³, and in the industrial production of nanoparticles⁴. The concept of particle trajectories has been used successfully to describe mixing and transport in turbulence^{3,5}, but issues of fundamental importance remain unresolved. One such issue is the Heisenberg–Yaglom prediction of fluid particle accelerations^{6,7}, based on the 1941 scaling theory of Kolmogorov^{8,9}. Here we report acceleration measurements using a detector adapted from high-energy physics to track particles in a laboratory water flow at Reynolds numbers up to 63,000. We find

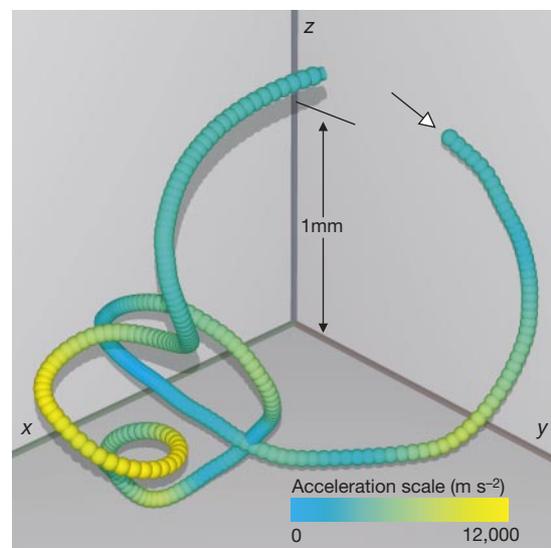


Figure 1 Measured particle trajectory. The three-dimensional time-resolved trajectory of a 46- μm -diameter particle in a turbulent water flow at Reynolds number 63,000 ($R_\lambda = 970$). A sphere marks the measured position of the particle in each of 300 frames taken every 0.014 ms ($\approx \tau_\eta/20$). The shading indicates the acceleration magnitude, with the maximum value of 12,000 m s^{-2} corresponding to approximately 30 standard deviations. The turbulence is generated between coaxial counter-rotating disks^{24,25} in a closed-flow chamber of volume 0.1 m^3 with rotation rates ranging from 0.15 Hz to 7.0 Hz, giving r.m.s. velocity fluctuation \bar{u} in the range $0.018 \text{ m s}^{-1} < \bar{u} < 0.87 \text{ m s}^{-1}$. Measurements are made in an 8- mm^3 volume at the centre of the apparatus where the mean velocity is zero and the flow is nearly homogeneous but not isotropic. As a result of a mean stretching of the flow along the propeller axis the r.m.s. fluctuations are one-third larger for the transverse velocity components than for the axial component. The energy dissipation was determined from measurements of the transverse second-order structure function D_{NN} and the Kolmogorov relation $D_{NN} = \frac{4}{3} C_1 (\epsilon r)^{2/3}$ with $C_1 = 2.13$ (ref. 26), ϵ the turbulent energy dissipation, and r the particle separation. The dissipation was found to be related to the r.m.s. velocity fluctuation by $\epsilon = \bar{u}^3/L$ with an energy injection scale $L = (71 \pm 7) \text{ mm}$. Using the definition of the Taylor microscale Reynolds number $R_\lambda = (15\bar{u}L\nu)^{1/2}$, the range of Reynolds numbers accessible is $140 \leq R_\lambda \leq 970$ (in terms of the classical Reynolds number $1,300 \leq \text{Re} \leq 63,000$). At the highest Reynolds number the system is characterized by Kolmogorov distance and timescales of $\eta = 18 \mu\text{m}$ and $\tau_\eta = 0.3 \text{ ms}$, respectively.

that, within experimental errors, Kolmogorov scaling of the acceleration variance is attained at high Reynolds numbers. Our data indicate that the acceleration is an extremely intermittent variable—particles are observed with accelerations of up to 1,500 times the acceleration of gravity (equivalent to 40 times the root mean square acceleration). We find that the acceleration data reflect the anisotropy of the large-scale flow at all Reynolds numbers studied.

In principle, fluid particle trajectories are easily measured by seeding a turbulent flow with minute tracer particles and following their motions with an imaging system. In practice this can be a very challenging task because we must fully resolve particle motions which take place on timescales of the order of the Kolmogorov time, $\tau_\eta = (\nu/\epsilon)^{1/2}$ where ν is the kinematic viscosity, ϵ is the turbulent energy dissipation and η is the Kolmogorov distance. This is exemplified in Fig. 1, which shows a measured three-dimensional, time-resolved trajectory of a tracer particle undergoing violent accelerations in our turbulent water flow, for which $\tau_\eta = 0.3$ ms. The particle enters the detection volume on the upper right, is pushed to the left by a burst of acceleration and comes nearly to a stop before being rapidly accelerated (at 1,200 times the acceleration of gravity) upward in a corkscrew motion. This trajectory illustrates the difficulty in following tracer particles—a particle's acceleration can go from zero to 30 times its root mean square (r.m.s.) value and back to zero in fractions of a millisecond and within distances of hundreds of micrometres.

Conventional detector technologies are effective for low Reynolds number flows^{10,11}, but do not provide adequate temporal resolution at high Reynolds numbers. However, the requirements are met by the use of silicon strip detectors as optical imaging elements in a particle-tracking system. The strip detectors used in our experiment (see Fig. 2a) were developed to measure particle tracks in the vertex detector of the CLEO III experiment operating at the Cornell Electron Positron Collider¹². When applied to particle tracking in turbulence (see Fig. 2b) each detector measures a one-dimensional projection of the image of the tracer particles. Using a data acquisition system designed for the turbulence experiment, several detectors can be simultaneously read out at up to 70,000 frames per second.

The acceleration of a fluid particle, \mathbf{a}^+ , in a turbulent flow is given by the Navier–Stokes equation:

$$\mathbf{a}^+ = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} \quad (1)$$

where p is the pressure, ρ is the fluid density, and \mathbf{u} is the velocity

field. In fully developed turbulence the viscous damping term is small compared to the pressure gradient term^{13,14} and therefore the acceleration is closely related to the pressure gradient.

Our measurement of the distribution of accelerations is shown in Fig. 3, where the probability density function of a normalized acceleration component is plotted at three Reynolds numbers. All of the distributions have a stretched exponential shape, in which the tails extend much further than they would for a gaussian distribution with the same variance. This indicates that accelerations many times the r.m.s. value are not as rare as one might expect, that is, the acceleration is an extremely intermittent variable. The acceleration flatness, shown in the inset to Fig. 3, characterizes the intermittency of the acceleration, and would be 3 for a gaussian distribution. These flatness values are consistent with direct numerical simulation (DNS) at low Reynolds number¹⁴ and exceed 60 at the highest Reynolds numbers.

The prediction by Heisenberg and Yaglom of the variance of an acceleration component based on Kolmogorov '41 theory^{8,9} is

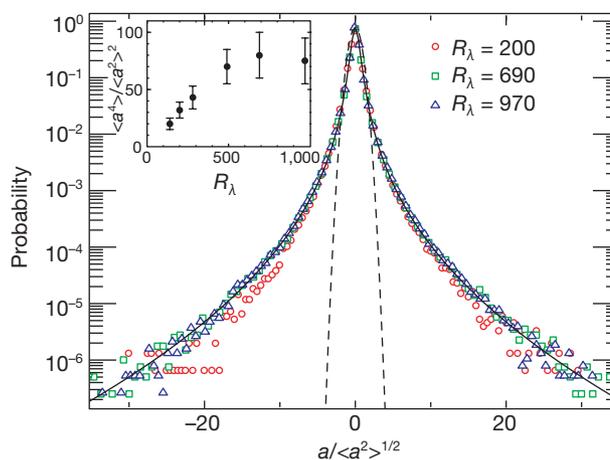


Figure 3 Acceleration distribution. Probability density functions of the transverse acceleration normalized by its standard deviation at different Reynolds numbers. The acceleration is measured from parabolic fits over $0.75\tau_\eta$ segments of each trajectory. The solid line is a parameterization of the highest Reynolds number data using the function $P(a) = C \exp(-a^2 / ((1 + |\alpha\beta|\sigma|^\gamma)\sigma^2))$, with $\beta = 0.539$, $\gamma = 1.588$ and $\sigma = 0.508$, and the dashed line is a gaussian distribution with the same variance. The inset shows the flatness of the acceleration distribution, $\langle a^4 \rangle / \langle a^2 \rangle^2$, evaluated using $0.5\tau_\eta$ parabolic fits) as a function of R_λ .

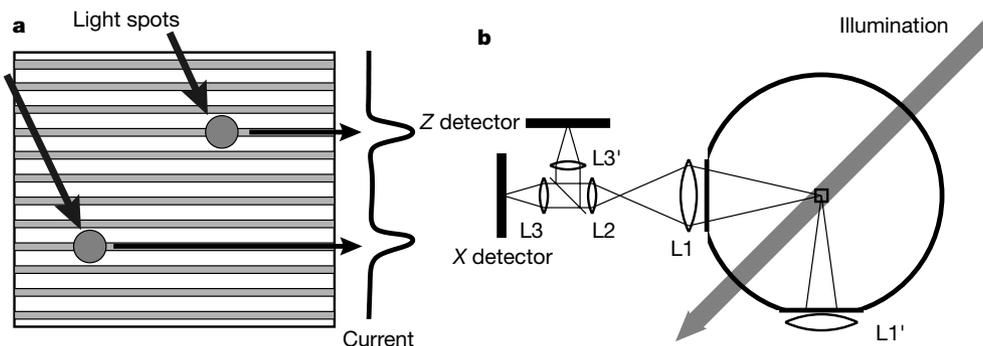


Figure 2 Apparatus. **a**, Schematic representation of the CLEO III strip detector¹², in which grey bars indicate sense strips which collect charge carriers freed by optical radiation. The 511 strips allow measurement of the one-dimensional projection of the light striking the detector. The detector may be read out 70,000 times per second. **b**, A combination of lenses (L1, L2, L3, L3') is used to image the active volume onto a pair of strip detectors which are oriented to measure the x and y coordinates. Another detector assembly may be

placed on the opposite port (L1') to measure y and z . The flow is illuminated by a 6 W argon ion laser beam oriented at 45° with respect to the two viewports. The optics image $(46 \pm 7)\text{-}\mu\text{m}$ -diameter transparent polystyrene spheres which have a density of 1.06 g cm^{-3} . Particle positions are measured with an accuracy of 0.1 strips, corresponding to $0.7\text{ }\mu\text{m}$ in the flow.

$$\langle a_i a_j \rangle = a_0 \epsilon^{3/2} \nu^{-1/2} \delta_{ij} \quad (2)$$

where a_0 is a universal constant which is approximately 1 in a model assuming gaussian fluctuations^{6,7,13,15}. However, DNS has found that a_0 depends on ϵ . Conventionally this is expressed in terms of the Taylor microscale Reynolds number, R_λ , which is related to the conventional Reynolds number by $R_\lambda = (15\text{Re})^{1/2}$ and is proportional to $\epsilon^{1/6}$. Using this notation, DNS results indicate $a_0 \approx R_\lambda^{1/2}$ for $R_\lambda < 250$ (ref. 14), with a tendency to level off¹⁶ as R_λ approaches 470.

Our measurement of the Kolmogorov constant a_0 is shown in Fig. 4 for eight orders of magnitude of scaling in acceleration variance. We find a_0 to be anisotropic and to depend significantly on the Reynolds number. The a_0 values for both components increase as a function of Reynolds number up to $R_\lambda \approx 500$, above which they are approximately constant. The trend in a_0 is consistent with DNS results in the range $140 \leq R_\lambda \leq 470$ (refs 14, 11, 17). However, the constant value of a_0 at high Reynolds number suggests that Kolmogorov '41 scaling^{8,9} is attained at higher Reynolds numbers. Owing to experimental uncertainties, weak deviations from the Kolmogorov '41 scaling^{8,9} such as the $a_0 \approx R_\lambda^{0.135}$ prediction of the Borgas multifractal model¹⁸ cannot be ruled out by our measurements.

The acceleration variance is larger for the transverse component than for the axial component at all values of the Reynolds number. This is shown in the inset to Fig. 4 where the ratio of the Kolmogorov constants for the axial and transverse acceleration components is plotted as a function of Reynolds number. The anisotropy is large at low Reynolds number and diminishes to a small value at $R_\lambda = 970$. This observation is consistent with recent experimental results which indicate that anisotropy persists to high Reynolds numbers^{19,20}.

In summary, our measurements indicate that the Heisenberg–Yaglom scaling of acceleration variance is observed for $500 \leq R_\lambda \leq 970$. At lower Reynolds number, our measurements are consistent with the nonuniversal scaling observed in DNS^{14,16}. Our measurements show that the anisotropy of the large scales affects the acceleration components even at $R_\lambda \approx 1,000$. It is impossible to say on the basis of these measurements if the

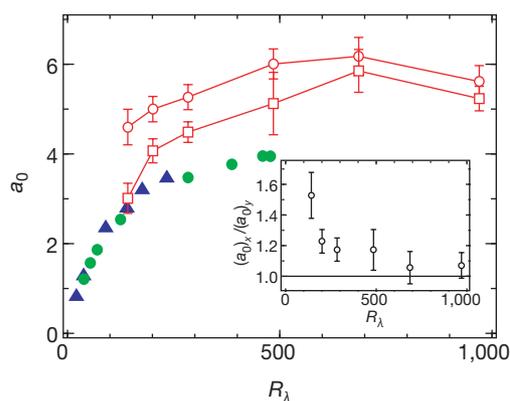


Figure 4 Acceleration a_0 as a function of R_λ . Open red circles indicate a transverse component and open red squares the axial component of the acceleration variance. Direct numerical simulation data is represented by blue triangles¹⁴ and green circles¹⁶. The error bars represent random and systematic errors in the measurement of the acceleration variance. There is an additional uncertainty of 15% in the overall scaling of the vertical axis for the experimental data due to the uncertainty in the measured value of the energy dissipation. The degree to which the 45- μm -diameter tracer particles follow the flow was investigated by measuring the acceleration variance as a function of particle size and density. The results, to be published elsewhere, confirm that the acceleration variance of the 46- μm particles is within a few per cent of the zero particle size limit. The inset shows the ratio of the a_0 values for transverse and axial components of the acceleration.

anisotropy will persist as the Reynolds number approaches infinity. We have found the acceleration to be a very intermittent variable, with extremely large accelerations often arising in structures such as the one shown in Fig. 1.

Our results have immediate application for the development of lagrangian stochastic models, some of which use a_0 directly as a model constant. These models are being developed and used to efficiently simulate mixing, particulate transport, and combustion in practical flows with varying Reynolds numbers^{3,21,22}. Our research also has surprising implications for everyday phenomena. For instance, a mosquito flying on a windy day (wind speed 18 km h⁻¹ and an altitude of 1 m) would experience an r.m.s. acceleration of 15 m s⁻². But given the extremely intermittent nature of the acceleration, the mosquito could expect to experience accelerations of 150 m s⁻² (15 times the acceleration of gravity) every 15 seconds. This may explain why, under windy conditions, a mosquito would prefer to cling to a blade of grass rather than take part in the rollercoaster ride through the Earth's turbulent boundary layer²³. □

Received 6 November 2000; accepted 3 January 2001.

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Acknowledgements

This research is supported by the Physics Division of the National Science Foundation. We thank R. Hill, M. Nelkin, S. B. Pope, E. Siggia, and Z. Warhaft for stimulating discussions and suggestions throughout the project. We also thank C. Ward, who assisted in the initial development of the strip detector. E.B. and A.L.P. are grateful for support from the Institute of Theoretical Physics at the University of California, Santa Barbara.

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